

CGC @ EIC

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Small x physics in the EIC era workshop, RBRC



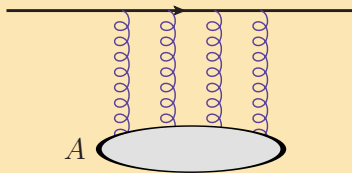
Outline

Aspects of CGC calculations for EIC processes

- ▶ Power counting
- ▶ Probes of saturation at EIC:
 - ▶ Evolution
 - ▶ Inclusive
 - ▶ Diffractive
 - ▶ Heavy quarks
 - ▶ Jets, azimuthal correlations
- ▶ EIC kinematics, experimental signals for saturation

Disclaimer: trying to give broad brush picture of status, no details . . .

Eikonal scattering off target of glue



How to measure small- x glue?

- ▶ Dilute probe through target color field
- ▶ At high energy interaction is eikonal

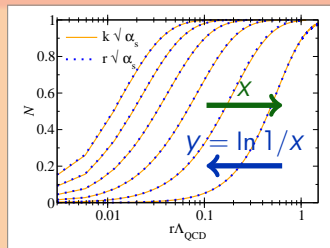
Eikonal scattering amplitude: Wilson line V

$$V = \mathbb{P} \exp \left\{ -ig \int^{x^+} dy^+ A^-(y^+, x^-, \mathbf{x}) \right\} \xrightarrow{x^+ \rightarrow \infty} V(\mathbf{x})$$

- ▶ Amplitude for color dipole

$$\mathcal{N}(r = |\mathbf{x} - \mathbf{y}|) = 1 - \left\langle \frac{1}{N_c} \text{Tr} V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$

- ▶ $1/Q_s$ is Wilson line \perp **correlation length**

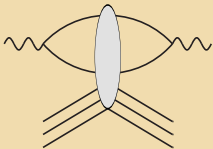


Power counting

Dilute-dense process at LO

Physical picture at small x

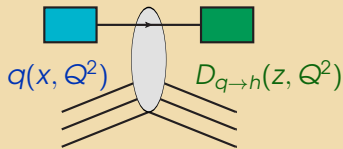
DIS



- ▶ $\gamma^* \rightarrow q\bar{q}$ dipole interacts with target color field
- ▶ Total cross section:
 $2 \times \text{Im-part of } \gamma^* \rightarrow \gamma^* \text{ amplitude}$

"Dipole model": Nikolaev, Zakharov 1991, Mueller
Fits to HERA data: e.g. Golec-Biernat, Wüsthoff 1998

Forward hadrons in pA



- ▶ q/g from probe: collinear pdf
- ▶ $|\text{amplitude}|^2 \sim \text{dipole}$
- ▶ Independent fragmentation
- ▶ Produced q/q p_T from target: $\sim Q_s$

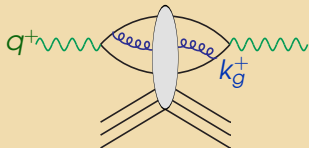
"Hybrid formalism"; Dumitru, Jalilian-Marian 2002

Both involve same dipole amplitude $\mathcal{N} = 1 - S$

Dilute-dense process at LL

Add one **soft** gluon: large logarithm of energy/x

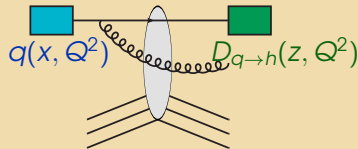
DIS



- ▶ Soft gluon: large logarithm

$$\int_{x_{Bj}} \frac{dk_g^+}{k_g^+} \sim \ln \frac{1}{x_{Bj}}$$

Forward hadrons



- ▶ Soft gluon $k^+ \rightarrow 0$: same large log
- ▶ Collinear gluon $k_T \rightarrow 0$:
DGLAP evolution of pdf & FF

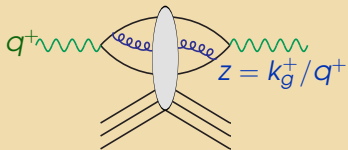
Dumitru et al 2005

Absorb large log into renormalization of target: **BK equation** Balitsky 1995, Kovchegov 1999

Dilute-dense process at NLO

Add one gluon, but **not** necessarily soft

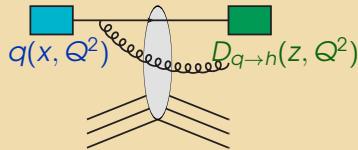
DIS



► DIS impact factor

Balitsky & Chirilli 2010, Beuf 2017

Forward hadrons



► NLO single inclusive

Chirilli et al 2011

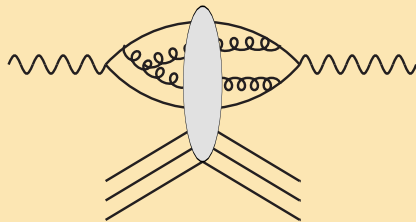
- Leading small- k^+ gluon already in BK-evolved target
- Need to **subtract** leading log from cross section:

$$\sigma_{NLO} = \int dz \left[\overbrace{\sigma(z) - \sigma(z=0)}^{\sigma_{\text{sub}}} + \overbrace{\sigma(z=0)}^{\text{absorb in BK}} \right] \quad z = \frac{k_g^+}{p_{\text{tot}}^+}$$

NLO to NLL

NLO evolution equation:

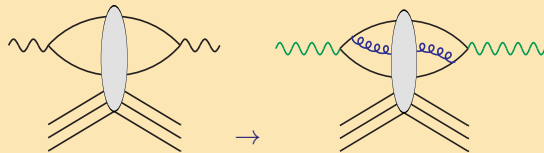
- ▶ Consider NNLO DIS
- ▶ Extract leading soft logarithm
- ▶ Lengthy calculation:
Balitsky & Chirilli 2007
- ▶ But additional resummations needed for practical phenomenology



(+ many diagrams at same order)

- ▶ $\alpha_s^2 \ln^2(1/x)$: two iterations of LO BK
- ▶ $\alpha_s^2 \ln 1/x$: NLO BK
- ▶ α_s^2 : part of NNLO impact factor (not calculated)

Summary: power counting



$$\sigma \sim \underbrace{\mathcal{O}(1)}_{\text{LO}} + \underbrace{\mathcal{O}(\alpha_s \ln 1/x)}_{\text{LL}} + \underbrace{\mathcal{O}(\alpha_s)}_{\text{NLO}} + \underbrace{\mathcal{O}(\alpha_s^2 \ln 1/x)}_{\text{NLL}} + \underbrace{\mathcal{O}(\alpha_s^2)}_{\text{NNLO}} + \underbrace{\mathcal{O}(\alpha_s^3 \ln 1/x)}_{\text{NNLL}}$$

So where are we?

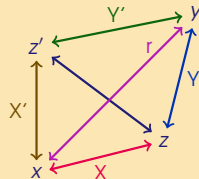
State of the art

Evolution

- ▶ Mean field limit = BK equation, $\left\langle \frac{1}{N_c} V^\dagger V \frac{1}{N_c} V^\dagger V \right\rangle \approx \left\langle \frac{1}{N_c} V^\dagger V \right\rangle \left\langle \frac{1}{N_c} V^\dagger V \right\rangle = S^2$

- ▶ NLO equation derived Balitsky, Chirilli 2008

$$\begin{aligned} \partial_Y S(r) = & \frac{\alpha_s N_c}{2\pi^2} \mathbf{K}_1 \otimes [S(X)S(Y) - S(r)] + \frac{\alpha_s^2 N_F N_c}{8\pi^4} \mathbf{K}_f \otimes S(Y)[S(X') - S(X)] \\ & + \frac{\alpha_s^2 N_c^2}{8\pi^4} \mathbf{K}_2 \otimes [S(X)S(z - z')S(Y') - S(X)S(Y)] \end{aligned}$$



- ▶ However, as written, plagued by instabilities from large transverse logarithms.

⇒ Need to resum Beuf 2014, Iancu, Triantafyllopoulos et al ~ 2015

In fact, **LO equation + resummation** can capture most NLO effects T.L., Mäntysaari 2016

⇒ State of the art in phenomenology

- ▶ NLO JIMWLK evolution: for Wilson lines V with “open index”

Balitsky, Chirilli 2013, Lublinsky, Mulian et al 2013, partially Grabovsky 2012, Caron-Huot 2015

- ▶ But so far in “Fock-Planck” form.
- ▶ To solve would want “Langevin” form, not available yet.

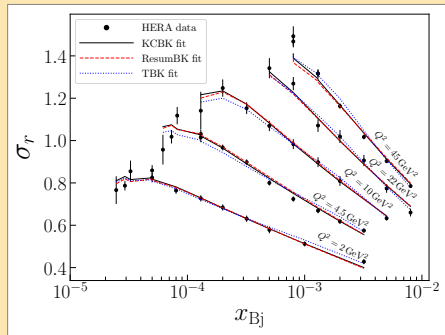
Inclusive DIS cross section

- ▶ Work in “dipole factorization,” first need $\gamma^* \rightarrow q\bar{q}$ wavefunction at NLO:

$$|\gamma^*\rangle = (1 + \dots)|\gamma^*\rangle_0 + \overbrace{\psi^{\gamma^* \rightarrow q\bar{q}}}_{1 \text{ loop}} \otimes |q\bar{q}\rangle_0 \\ + \psi^{\gamma^* \rightarrow q\bar{q}g} \otimes |q\bar{q}g\rangle_0 + \dots$$

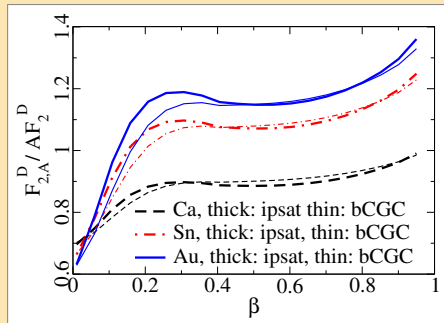
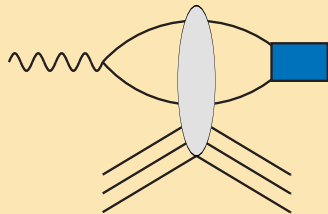
Beuf 2016, T.L., Paatelainen 2017

- ▶ Then square, cancel divergences, factorize into BK
- ▶ First fits to HERA H. Hänninen, T.L., H. Mäntysaari 2020
- ▶ Why fits? Factorization scheme details, BK resummations . . .



Diffraction DIS

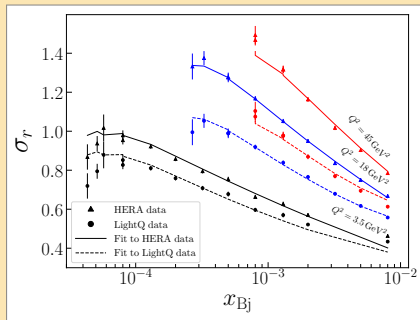
- ▶ Smoking gun for saturation
- ▶ Quadratic in “gluon density”
- ▶ NLO Calculation in dipole picture:
use same $\gamma \rightarrow q\bar{q}$ LCWF
- ▶ Then project to final state
 - ▶ Vector meson: need wavefunction
 - ▶ Jets: calculable
 - ▶ Total diffraction (F_2^D): calculable
- ▶ Status: NLO calculations exist
e.g. Boussarie et al 2014, Caucal et al 2021
but phenomenology still at the level of LO
- ▶ More in other talks: [Iancu](#), [Mäntysaari](#)



Heavy quarks

Elephant in the room: simultaneous F_2 & F_2^C difficult even at LO

- ▶ Until recently NLO only for $m = 0$
- ▶ Charm is large fraction of F_2
- ▶ Beuf, Hänninen, T.L., Mäntysaari 2021 :
Can fit interpolated “light quark-only” data with NLO dipole picture
- ▶ Result: good χ^2 , but parameters unnatural
- ▶ Lesson: F_2 is not ideal for dipole picture
(known: aligned jet configurations)
 $\Rightarrow F_L$ and F_2^C for more reliable probes.

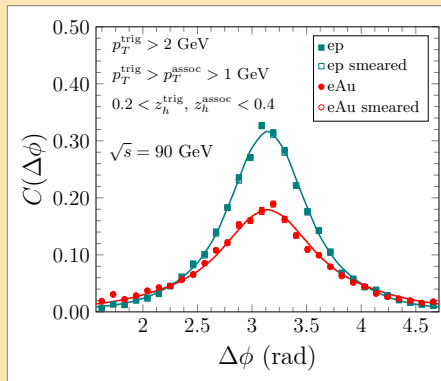


Luckily, situation is changing

- ▶ $\gamma^* \rightarrow q\bar{q}$ 4 1-loop wavefunctions with $m_q \neq 0$: Beuf, T.L., Paatelainen 2021 and t.b.p
- ▶ First applications: exclusive vector mesons (with NRQCD) Mäntysaari, Penttala

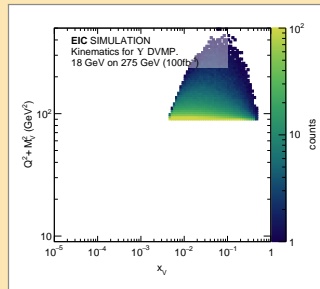
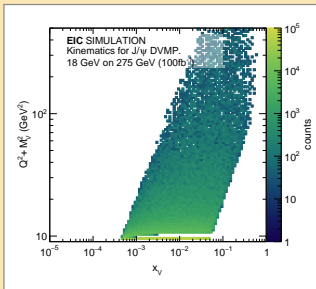
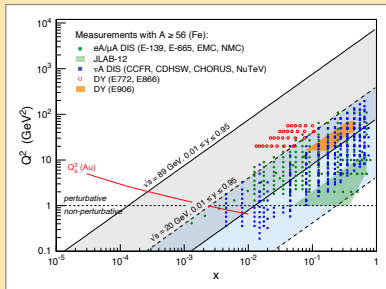
Jets, correlations

- ▶ Q_s is a transverse momentum scale
 \Rightarrow generically expect effects in \mathbf{p} correlations
- ▶ Dihadron correlations, angular modification
But how realistic Hatta, Xiao, Yuan, Zhou 2021 ?
- ▶ Wigner distribution Yuan



Comment on kinematics

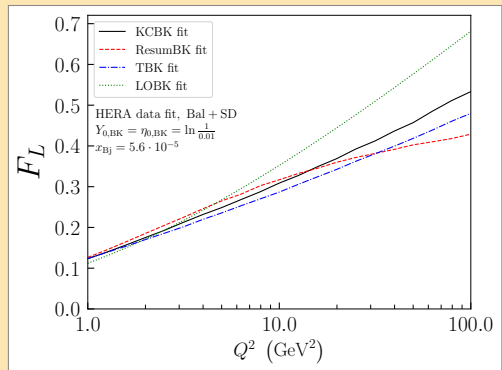
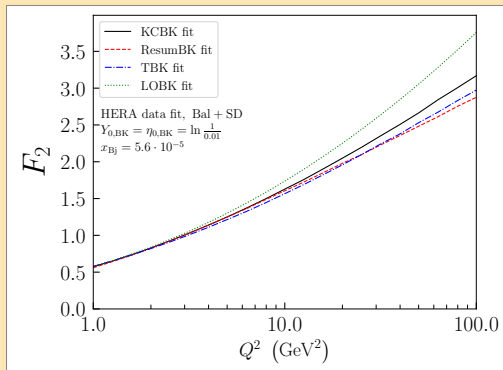
EIC kinematics: how low do you go?



- ▶ Charm: $x \gtrsim 10^{-3}$, beauty $x \gtrsim 10^{-2}$
- ▶ For $x = 0.01$, $Q^2 \lesssim 5 \text{ GeV}^2 \Rightarrow$ Not huge leverarm in $\ln 1/x$, $\ln Q^2$
- ▶ EIC YR starts talking about jets at $p_T > 5 \text{ GeV}$. Lower than pp , but not low x
 \Rightarrow small- x calculations need to be for hadrons, not jets

E.g. different versions of resummed BK

Beuf, Hänninen, T.L., Mäntysaari 2021



Differences between versions of resummed BK small, even at LHeC kinematics

(Once adjusted to same HERA data)

- + predictions are robust
- will not distinguish experimentally
- Transient effects, factorization scheme important!

Conclusions

- ▶ Calculations of many EIC processes advancing to higher order in α_s
in saturation region — but much work to do yet.
- ▶ Formula for cross section \neq numbers on plot:
factorization, resummation schemes
- ▶ Personal opinion:
Intricate details of evolution, jets at small x hard to access in EIC kinematics
- ▶ Goal (?) : coherent, consistent description of variety of processes:
inclusive, diffractive, exclusive processes, light, heavy quarks

Apologies for very biased selection of references, could not cover everything!